Static Program Analysis Using Type and Effect Systems

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Static Program Analysis

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An efficient way to predict runtime behaviors of programs, including software defects.

Typical approaches

- Data flow analysis
- Abstract interpretation
- Inference-based analysis

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Typical approaches

- Inference-based analysis
  - Type and effect systems:
    - Simplicity
    - Effectiveness
    - Correctness can be formally proved

Publications

- Jens Palsberg, UCLA — Type Based Analysis and Applications
  http://www.cs.ucla.edu/~palsberg/tba/
- Flemming Nielson, Hanne R. Nielson, Chris Hankin — Principles of Program Analysis
- Bjarne Steensgaard, MSR — Points-to Analysis in Almost Linear Time
- Chandrasekhar Boyapati and Martin Rinard — A Parameterized Type System for Race-Free Java Programs

and 50+ articles more, mostly theoretical...

Type Systems (Type Theory)

Typing relation

$$\Gamma \vdash e : \tau$$

Where:

- Γ is the typing environment
- e is the expression
- τ is the type

Type Soundness

«Well typed programs do not go wrong» [a]

[a] Benjamin C. Pierce, Types and Programming Languages

Type Systems (Type Theory)

Example

$$e ::= v \mid e + e \mid \text{if } e \text{ then } e \text{ else } e \mid \text{isZero } e$$

$$v ::= \text{True} \mid \text{False} \mid 0 \mid 1 \mid \cdots$$

$$\tau ::= \text{Bool} \mid \text{Int}$$

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$$\Gamma \vdash \text{True} : \text{Bool} \quad \Gamma \vdash \text{False} : \text{Bool} \quad \Gamma \vdash 0 : \text{Int} \quad \cdots$$

$$\frac{\Gamma \vdash x_1 : \text{Int} \quad \Gamma \vdash x_2 : \text{Int}}{\Gamma \vdash (x_1 + x_2) : \text{Int}}$$

$$\frac{\Gamma \vdash x : \text{Int}}{\Gamma \vdash (\text{isZero } x) : \text{Bool}}$$

$$\frac{\Gamma \vdash c : \text{Bool} \quad \Gamma \vdash x_1 : \tau \quad \Gamma \vdash x_2 : \tau}{\Gamma \vdash (\text{if } c \text{ then } x_1 \text{ else } x_2) : \tau}$$

Type Systems for Static Analysis

Type system:

- Target language
- Types
- A set of inference rules

Typing relation

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Type Completeness

If a statement e has a type (i.e. is valid) in the analyzed language, then it should be also valid in the target language.

Example

Target Language

$$e ::= \text{Const} \mid \text{Create} \mid \text{Free } e \mid \bigcap e_1\ e_2 \mid \text{Return } e \mid e_1 := e_2$$

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or

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or

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or even

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- Also in Java:
  - `class B >: class A <=> A extends B`

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- Also in Java:
  - The «reference» part of typesystem is a complete lattice
  - The top element is `Object`
  - The bottom element is `null`
- And in Scala:
  - The whole typesystem is a lattice
  - The top element is `Any`
  - The bottom element is `Nothing`

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- ... or an upper semilattice?
- ... or a lower semilattice?

Example: subtypes

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Types

$$\tau ::= \text{def} \mid \text{undef} \mid \text{maydef}$$

$$\text{maydef} \sqsupseteq \text{def} \qquad \text{maydef} \sqsupseteq \text{undef}$$

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- Detection of single defect

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- Detection of single defect
- What if the defect is a false one?

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$$\psi \text{ is the «natural» type}$$

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$$\mathit{bool}_{\text{def}}, \quad \mathit{int}_{\text{undef}}, \quad \mathit{etc}.$$

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We also may put annotations on top of existent analysis.

Type and Effect Systems

Type and Effect System:

- Target Language
- Types
- A set of inference rules (for both types and effects)
- The partial ordering on types

Typing relation

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Where: Γ is the type environment

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$$\frac{\Gamma \vdash e : \text{undef} \,\&\, \varphi_0}{\Gamma \vdash \text{Return } e : \text{undef} \,\&\, (\varphi_0 \cup \{\mathtt{ERR}\})}$$

$$\frac{\Gamma \vdash e : \text{maydef} \,\&\, \varphi_0}{\Gamma \vdash \text{Return } e : \text{undef} \,\&\, (\varphi_0 \cup \{\mathtt{ERR}\})}$$

Getting more complicated...

- Special polymorphism
  - Type families $\forall a.a \to a$
  - Existential types $\exists b.a \to b$
- Complex effects (depending on types)
- Subeffects
- Dependent and recursive types $A = A \to A$
- Subeffecting
- etc.

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Subtyping

$$\{\nu : \tau | X(\nu)\} \sqsupseteq \{\nu : \tau | Y(\nu)\}$$
$$=$$
$$\forall \nu : X(\nu) \Rightarrow Y(\nu)$$

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- Well, as functional as we can :-(

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Substitution & Unification

- We have an equation: $X = Y$

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  - Theorem provers: HOL, Coq, etc.
  - SMT-solvers: Z3, Alt-Ergo, Yices, etc.

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- Or is it?

Choosing program model

- Designing our own model
- Modifying an existent one
  - AST — abstract syntax tree
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  - DDG — data dependency graph
  - SSA — static single assignment form

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Data dependency info

Phi-functions

Retrieving the SSA

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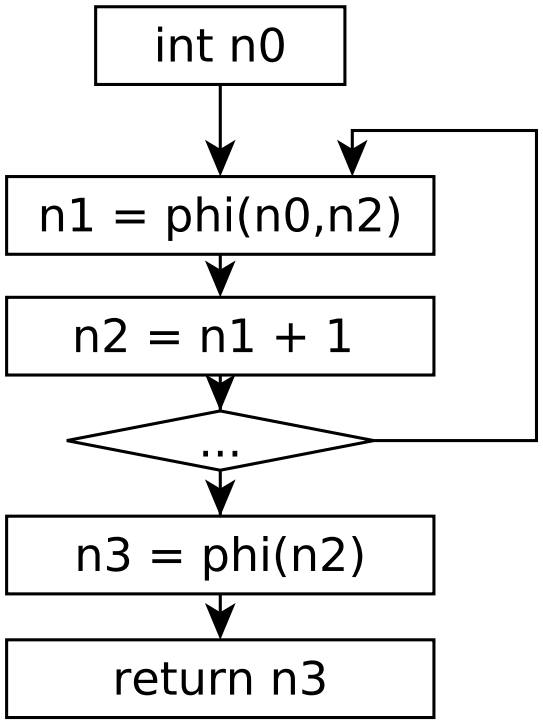
llvm-parser

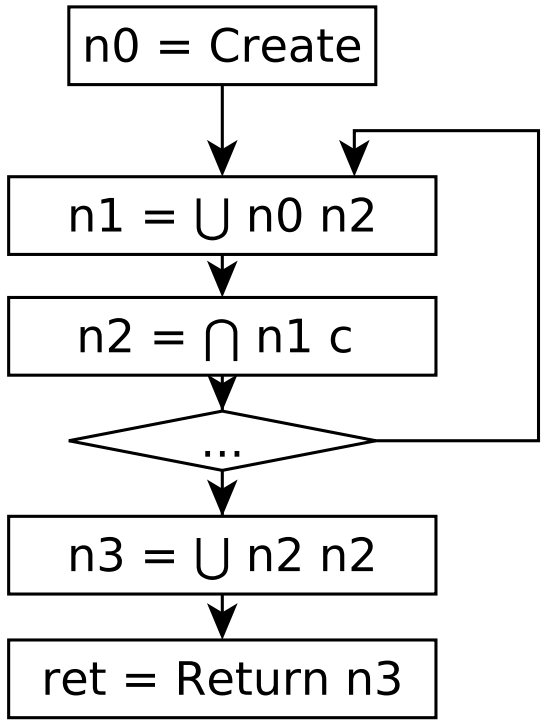
- Standalone parser of LLVM bitcode files
- Retrieves the model from files and makes it more suitable for typing
- Keeps the metadata info

How this works

C ⇒ SSA ⇒ Target language

```
int n;
do {
    n++;
} while (...);
return n;
```





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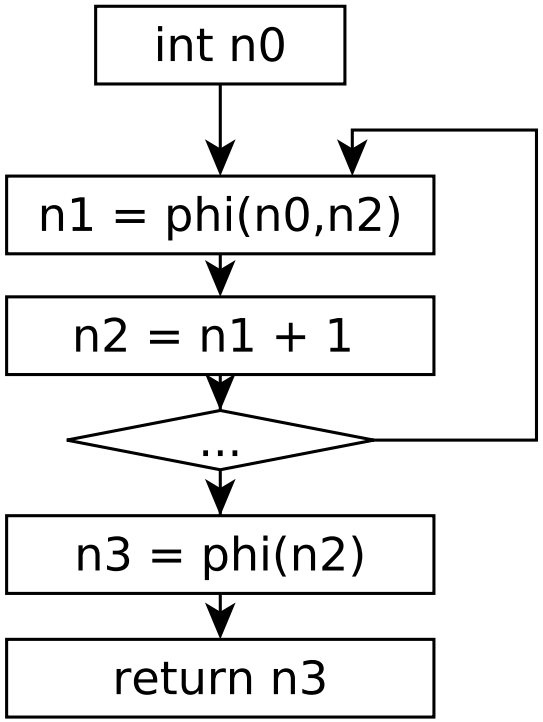

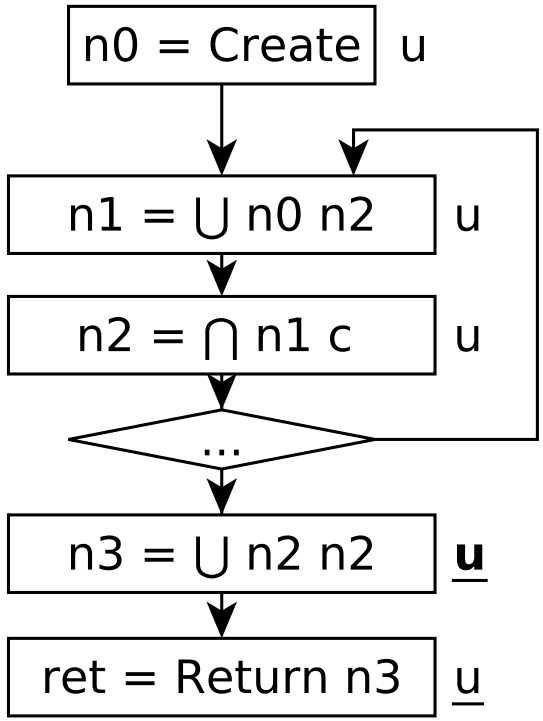

ERR!

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For a mathematical formula $X_1 * X_2 + X_3 = 500 \vee X_4 = 2$ what are X_K that evaluate it to true?

Solving type ambiguity

Using SMT-solver

1. Convert rules to first-order logic theorems
2. Find the correct solution with the minimal number of effects

Drawbacks

No guarantees and need to encode everything (types, effects, rules, etc.)

SBV library[1]

Tools — SBV library and its EDSL:

```
rules Constant = definition DDefined
rules (Assign val exp) =
    λ x e s → let t =  s ⊢ exp in x ≡ t
rules (JoinOr v0 v1) =
    λ x e s → let t0 = s ⊢ v0; t1 = s ⊢ v1 in
              x `pge` t0 ∧ x `pge` t1
rules (Return v) =
    λ x e s → let t =  s ⊢ v in
        if_ (DUndefined `typeEq` t ∨ DMaybeDefined `typeEq` t)
            (e `effect` 0)
```

[1] https://github.com/LeventErkok/sbv

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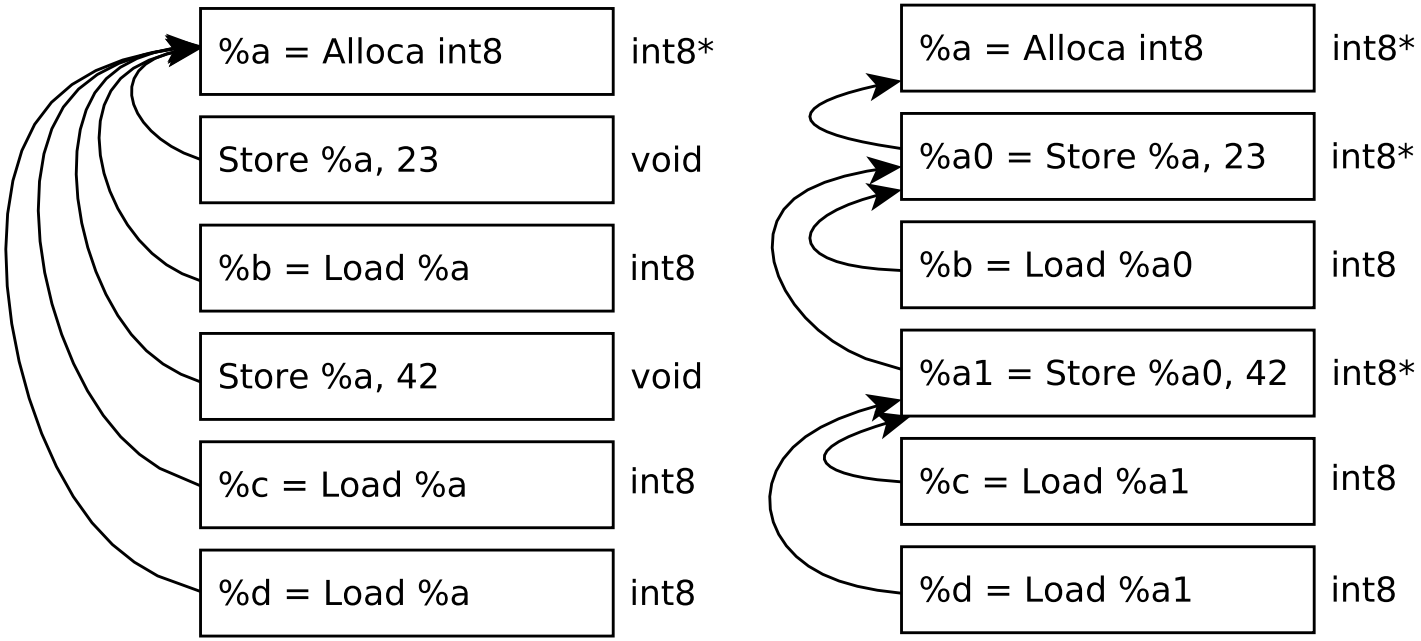

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- Stores and Loads
- Which instructions point to the same memory cell?

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Pointer versions!

How to make it?

- Stores and Loads — LLVM already has them!
- Which instructions point to the same memory cell? — need pointer analysis

Pointer analysis

- Alias Analysis — does the pointer A point to the same location as pointer B does?
- Point-to Analysis — model the heap as some kind of graph

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What's the difference?

- It's almost no difference really
- Context-insensitive analysis is NP-complete :)
- Context-sensitive analysis is undecidable :)
- Our project provides a very naive implementation of AA

Problems

- No working general solution :)

THE END